1. Santa Claus has $n \geq 2$ different gifts. He distributes these gifts in sacks (each sack can contain from 1 to n items) and puts the sacks with gifts around the Christmas tree (only the content of the sacks and their ordering on the circle around the tree are important). There are $E(n)$ ways of doing this with an even number of sacks and $O(n)$ ways with an odd number of sacks. Prove that $E(n) = O(n)$.

(For example, if there are three gifts A, B, C, then all possible variants are A-(BC), B-(AC), C-(AB); (ABC), A-B-C, A-C-B).

2. Given a natural number n, consider the function

$$f(x, y) = x^n + x^{n-1}y + x^{n-2}y^2 + \cdots + xy^{n-1} + y^n$$

of two real variables. Find the minimal number k for which there exist functions $g_1, \ldots, g_k, h_1, \ldots, h_k$ of one real variable such that

$$f(x, y) = \sum_{i=1}^{k} g_i(x)h_i(y).$$

3. At a point O there is a star with three planets revolving around it. The planets have circular orbits centered at O in different two-dimensional planes and constant pairwise different angle velocities. Is it true that there is always a moment of time at which the angles between the rays from O to the planets are all at least $90°$?

4. A river falling into a sea forms a delta which is a system of branches consisting of channels without inner intersections.

(a) Suppose that in a given delta there are exactly n different (i.e. differing by at least one channel) routes down the stream. Prove that there are at least $3 \log_3 n$ channels in this delta.

(b) Prove that there is a function $f(n)$ satisfying the condition

$$\lim_{n \to \infty} \frac{f(n)}{\log_3 n} = 3$$

and such that for every n a delta can exist with at most $f(n)$ channels admitting precisely n different routes down the stream.

5. Prove that for every odd prime number p there is a natural number n such that the congruence $n^5 + n^4 - 3 \equiv x^2 \pmod{p}$ has no solutions.