1. Given a straight line and a circle in the plane, on every chord of this circle parallel to the given line we construct a circle having this chord as a diameter. Which set is formed by all the circles constructed?

2. A real square matrix (a_{ij}) of order $n \geq 2$ is called narrow if $a_{ij} = 0$ whenever $1 \leq i, j \leq n - 1$. Suppose that four distinct narrow matrices A, B, C, D belong to the same straight line in the space of all matrices and that $A^4 = B^5 = C^6 = 0$.

Prove that $D^7 = 0$.

3. We are given two triangles Δ_1, Δ_2 with the corresponding side lengths $a_1 \leq b_1 \leq c_1$, $a_2 \leq b_2 \leq c_2$. Prove that

(i) there is a triangle Δ with side lengths $\sqrt{a_1^2 + a_2^2}$, $\sqrt{b_1^2 + b_2^2}$, $\sqrt{c_1^2 + c_2^2}$;

(ii) $S(\Delta) \geq S(\Delta_1) + S(\Delta_2)$, where $S(\Delta)$ is the area of the triangle Δ, and that the equality is possible only for similar triangles.

4. At the meeting of the round table "Math and Magic", where $n \geq 3$ persons are sitting at a round table, a magician and his assistant enter and have a short conversation, after which the magician leaves the room. His assistant announces that a trick will be shown. The participants of the meeting take seats around the table in an arbitrary order and then one of them leaves the table. The assistant asks one of the two neighbors of that participant to leave his place too. The magician returns to the room, looks at the two standing persons and $n - 2$ sitting and announces the order in which these two have been sitting. For which n such a trick is possible?

5. Let $f\colon \mathbb{R}^n \to \mathbb{R}^1$ be a homogeneous polynomial and $f(U + a) \subset [-1, 1]$ for some $a \in \mathbb{R}^n$, where U is a ball centered at the origin.

(i) Prove that $f(c \cdot U) \subset [-1, 1]$ if $c = \frac{1}{6}$.

(ii) For which $c > 0$ the assertion is valid independently of n, f, a, and U?