1. Prove that there is a homogeneous polynomial $f(x, y, z) \neq 0$ such that if in a triangle with side lengths a, b, c the angle opposite to the side a is 2017 times larger than the angle opposite to the side b, then $f(a, b, c) = 0$.

2. Prove that every rational number can be represented in the form $x^4 + y^4 - z^4 - t^4$ with rational numbers x, y, z, t.

3. Find all continuous functions f on $[0, 1]$ satisfying the identity

$$f(x) = (1 - x)f(x/2) + xf(x/2 + 1/2) \quad \forall x \in [0, 1].$$

4. Suppose we are given pairwise different points $x_1, \ldots, x_n, y_1, \ldots, y_n$ in $\mathbb{R}^d$ such that $d \leq 2n - 2$,

$$|x_1 - y_1| = |x_2 - y_2| = \cdots = |x_n - y_n| = a,$$

and all other distances between different points in this collection equal some number b. Prove that all these points belong to an n-dimensional affine subspace in $\mathbb{R}^d$.

5. Suppose we are given two mappings $F, G \colon \mathbb{R}^n \to \mathbb{R}^n$ such that $\|F(x) - F(y)\| \leq \|x - y\|/2$ and G is continuous and has the property that the image under G of every closed ball contains the ball of the same radius centered at the image of the center. Prove that there is a point p such that $F(p) = G(p)$.

6. Let us consider all Young diagrams contained in the rectangle $a \times b$ (that is, having at most a rows and at most b columns). For every such diagram, we sum the lengths of all hooks of its boxes. Prove that the total sum of the obtained numbers equals

$$\frac{(a + b + 1)!}{3!(a - 1)!(b - 1)!}.$$

(A Young diagram in the rectangle $a \times b$ is a set of boxes that along with every its box contains all boxes below and to the left of it. The hook of a box in a Young diagram consists of this box and all boxes of the diagram strictly under it and strictly to the right of it. The length of a hook is the number of boxes it contains.)