

1. Let M be the collection of all subsets of cardinality $4n$ in a set of cardinality $9n$. Is it true that for arbitrary sets $A_1, \dots, A_9 \in M$ there is a set $B \in M$ such that the cardinality of $B \cap A_k$ is less than $16n/9$ for each k ?

2. Does there exist a random variable ξ with integer values such that the probability of the event that ξ is divisible by n equals $1/n$ for every natural number n ?

3. Find all prime numbers p for which the following assertion is true: “if a matrix of size 2015×2015 over a field with p elements has the property that its rank decreases after an arbitrary nonzero element is replaced by zero, then every its row contains at most one nonzero element.”

4. An airplane is flying along a plane smooth bounded closed contour (without self-intersections) with its proper velocity that is constant in absolute value. The duration of the flight is T . Now a constant wind appears, and the airplane velocity must have a component against wind (at least with equal absolute value). Prove that the duration of the flight (with the resulting velocity $v + w$) round this contour is minimal when there is no wind.

5. A sequence of functions f_n on $[0, 1]$ is given. Is it true that one can always find a sequence of numbers $\varepsilon_n > 0$ such that $\lim_{n \rightarrow \infty} \varepsilon_n f_n(x) = 0$ for every $x \in [0, 1]$?